In their paper “the relativistic Sagnac effect: two derivations” Guido Rizzi and Matteo Luca discuss the Sagnac effect in a very lucid way, and convincingly argue that the effect is in no conflict at all with special relativity; on the contrary, the fact that the effect exists also in the case of matter waves can only be understood within the framework of relativity theory and thus supports this theory.

In their section 3.4.1 Rizzi and Ruggiero comment on a derivation of the Sagnac effect that was published by Gerard Nienhuis and myself; they invited me to discuss their comments.
The derivation by Nienhuis and myself considers the local comoving inertial frames of reference (with Einstein synchronization) along the rim of the rotating disk. In these frames the two beams have opposite velocities; the magnitude of these velocities is the same in all such frames along the rim. The time interval $d\tau$ that the beams need to transverse an infinitesimal distance, as judged from the local inertial systems, corresponds to a time dt in the laboratory system that can be found via the Lorentz transformation. Gluing all the local inertial systems together, it is possible to derive a relation between the lab times needed by the two beams to make a roundtrip, and the total time accumulatively measured in all the local inertial systems---see formula (34) of Rizzi and Ruggiero.
Rizzi and Ruggiero now say that two “hypotheses” play a role in the remainder of our argument. I have difficulty with this terminology: in my opinion there are no hypotheses involved, but only the use of data that are already inherent in the relativistic description of the situation. The first concerns the length of the circumference of the disk. Given the fact that the radius is R, the formula for this length is a deductive consequence of relativity theory. It is true that this formula is not explicitly used by Rizzi and Ruggiero themselves, but it is nevertheless a deductive consequence within their approach as well. The second “hypothesis” concerns the total times used by the two beams to make their roundtrips, as measured in the many consecutive comoving local inertial frames that “follow” the beams. Because the speeds of the two beams are equal in all these frames, and all distances are also the same, these two times must be equal. Again, it seems to me that this is a matter of logic and not an additional assumption.

There perhaps is confusion here because Rizzi and Ruggiero seem to use the same symbols $\tau_{\pm}$ for something else, namely times measured by **one** clock fixed to the rim of the disk.

Finally, Rizzi and Ruggiero comment on the comparison we made between the Sagnac effect and the twin effect; we wrote that the Sagnac effect can be regarded as a variation **on** (not “of”) the twin phenomenon. The idea was that in the twin effect proper times are different and coordinate times (on arrival) the same, but that this is reversed in the Sagnac case (the proper times are taken along the wordlines of the particles in the beams). It is a variation on a certain theme, like in music.